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# Time-Domain Electrical Impedance Tomography by Numerical Analysis of the Step Response

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**Abstract.** This work describes the theoretical basis of an electrical impedance tomography imaging system based on numerical analysis of the step response. Its novelty relies on the use of time domain for rendering the tomographic images. Following the injection of a Heaviside-step current through two electrodes, the voltage-response is measured on all couple of electrodes according to the neighbouring strategy; this process is repeated on every pair of consecutive electrodes. Based on the measurements, a tomographic image is reconstructed using the Gauss-Newton-Raphson algorithm. We tested the technique by simulating two representative circuits: one symmetrical pseudo-isotropic and one pseudo-anisotropic in AC, while both pseudo-isotropic at DC. The time-domain reconstructed images show the second network's pseudo-anisotropy while allowing the system to show its tendency to pseudo-isotropy when the time elapses towards DC-steady-state. This novel technique for reconstructing electrical impedance tomographic images may shed new light on sensing slight differences in tissues while being fast and low-cost.

## 1. Introduction

Electrical impedance measurements can characterize various living tissue (in-vivo and in-vitro), biological components, and suspensions. Within limits regarding bioelectrical impedance and frequency, living tissues and biological components can be considered linear and time-invariant (LTI) systems of lumped elements. As research progresses, a great variety of applications will be developed based on bioimpedance and its monitoring, due to its lack of adverse effects and low cost. As a reasonable alternative to costly and hazardous procedures, such as nuclear medicine, electrical bioimpedance will fulfill clinical measurement and monitoring necessities [1]. One such clinical desideratum is the continuous edema-extent monitoring by electrical impedance tomography (EIT) of intensive care patients [2, 3].

Although computing power is a decreasing limitation as quicker electronic components become available, presently the image reconstruction must be performed *in-toto* before any tomographic result can be shown. Placement of the electrodes strap around the patient's thorax needs a quick feedback to guide the maneuver, a feedback that could consist of a tomographic slice of increasing definition as time passes. This incremental tomographic reconstruction is not possible with present algorithms [4]. We have therefore a clear motivation to look for a method that would give a fast preliminary tomographic reconstruction.



The step-response method, used for the analysis and characterization of LTI systems has found use in many control theory applications [5]. Being a fast method for systems analysis, we set ourselves the goal to explore its use for EIT image reconstruction. While classic frequency response analyses require several full periods of sinusoidal signals to be injected into the system for each frequency of interest, the step-response signal already carries the system's spectral information in a single sweep [5,6]. The extension in time of the Heaviside step will determine the lowest frequency analyzed by the method. Step-response analysis is much less time-consuming than frequency response analysis [7]. In addition, the step-response analysis seems to fulfill our goal of immediate coarse result and finer result later. However, digital step-response analyses are much more susceptible to system-intrinsic non-idealities [7,8]. Fitting the objective sampled signal to well-known analytical expressions may help to override such drawbacks [9,10].

Among the accepted analytical models of electrical impedance, we find Debye (for non-DC-conducting materials) and Lapicque and Cole (for DC-conducting materials) [9,10]. In 2017, David et al. [11] demonstrated that using a finite combination of different Debye and Lapicque models could fully describe the electrical bioimpedance of Red Blood Cells suspensions.

A recent work by Zhang et al., from 2020 [12], presented a novel method for estimating the full Cole impedance model by analyzing the impedance response to a DC-biased sinusoidal excitation. This method is the first able to estimate the full Cole spectral information from a signal combining a single frequency and a DC step.

Our work focuses on using the voltage obtained as a response to a step excitation, without extracting the full impedance model in Laplace domain. In this paper, we propose to harness the versatility of the analysis of the step-response towards developing a fast and reliable EIT system able to perform in the time domain.

## 2. Analytical methods and approach

### 2.1. Continuous-time analysis

The simplest electric circuit representing impedance analysis using step-response is a step current source connected to the impedance-under-test, and the voltage that drops is measured. In this approach, we neglect the parasitic effects of the cables and connectors. Those effects need to be corrected off-line after proper calibration with a known load.

Measuring the output voltage generated on the impedance allows studying the mathematical connection between the impedance and the current step. Since the whole system (including the impedance) is LTI, the expression of the output voltage is a convolution of the time-domain impedance function  $z(t)$  (defined as the inverse Laplace transform of the impedance in the frequency domain) with the step current.

$$\begin{aligned} i(t) &= I_0 \cdot u(t) \\ V_{impedance}(t) &= z * i(t) \end{aligned} \quad (1)$$

where  $I_0$  is the current intensity after the step,  $u(t)$  is the Heaviside function in time, and  $*$  is the convolution operator.

Naturally, the sampled voltage on the impedance will carry noise resulting from electromagnetic interference, jitter of the sampler, quantization noise, and other sources. Hence, fitting the sampled voltage to known analytical expressions (see Section 2.3) will render cleaner signals to be used for the tomographic reconstruction (see Section 2.4).

### 2.2. Debye, Lapicque, and Cole impedance models

Two basic impedance models are mainly accepted: Lapicque and Debye [9,13]. The following equation shows their expressions in the Laplace domain. Both models present a resistor ( $R_d$ ) and a capacitor ( $C_d$ ) in parallel, connected in series to a resistor  $R_i$  (Lapicque) or a capacitor  $C_i$  (Debye). Equation (2) presents the impedances expressions in the Laplace domain.

$$\begin{aligned} Z_{Lapicque}(s) &= R_i + \frac{R_d}{1+s \cdot R_d C_d} \\ Z_{Debye}(s) &= \frac{1}{s \cdot C_i} + \frac{R_d}{1+s \cdot R_d C_d} \end{aligned} \quad (2)$$

In general, the Debye model is used for describing interface-polarization phenomena, typically explained by the Maxwell-Wagner model [9], where at least one of the interfaces has no DC conductivity.

The Cole model extends the Lopicque model and cannot be described in polynomial terms of integer exponents [14]. Its application to electrical impedances renders equation (3), showing its expression in the Laplace domain.

$$Z_{CC}(s) = R_\infty + \frac{R_\Delta}{1+s^\alpha \cdot R_\Delta C} \quad (3)$$

The time/frequency modeling of Cole impedances leads to the use of fractional derivatives in time. Fractional derivatives are used to evaluate the regularized values of finite-part integrals; thus, they can be immediately used for the numerical evaluation of fractional-order Laplace functions. In 1903 and 1904, Mittag-Leffler presented the basis for the analytical development of the (later) called Mittag-Leffler's function, used to calculate the inverse Laplace-transform of a function with fractional exponent in  $s$  [15–18].

### 2.3. Step-responses in the time domain

By applying the corresponding inverse Laplace transforms, the step-response functions in time (for Lopicque and Debye impedances) present the forms given by (4)

$$\begin{aligned} V_{Lapicque}(t) &= I_0 \cdot \left[ (R_i + R_d) - R_d \cdot e^{-t/\tau} \right] \cdot u(t) \\ V_{Debye}(t) &= I_0 \cdot \left[ C_i \cdot t + R_d \cdot (1 - e^{-t/\tau}) \right] \cdot u(t) \end{aligned} \quad (4)$$

where  $\tau = R_d \cdot C_d$ .

The step-response analytical expression for the Cole impedance model is obtained by performing a partial fractions expansion of the expression given in (5).

$$V_{CC}(s) = \frac{R_\infty}{s} + \frac{R_\Delta}{s} - \frac{R_\Delta \cdot s^{\alpha-1}}{s^\alpha + \frac{1}{R_\Delta C}} \quad (5)$$

Mittag-Leffler's function [15–21] describes the inverse Laplace transform of (10) and is defined by equation (6).

$$E_\alpha(kt^\alpha) = \sum_{n=0}^{+\infty} \frac{(kt^\alpha)^n}{\Gamma(\alpha n + 1)} \quad (6)$$

where  $\Gamma$  is the Gamma function (extension of the factorial function),  $\alpha$  is the Cole exponent, and  $k$  is a coefficient of  $t$  as presented in equation (7).

Thus, the time-domain step function of a Cole impedance is given by

$$V_{CC}(t) = I_0 \cdot \left[ R_\infty + R_\Delta - R_\Delta E_\alpha \left( -\frac{t^\alpha}{R_\Delta C} \right) \right] \cdot u(t) \quad (7)$$

It is worth noting that in 2011, Freeborn et al. [22] proposed a pseudo-algorithm for efficient fitting and parameter extraction of a Mittag-Leffler's function.

### 2.4. Tomographic image reconstruction

The image reconstruction is done using the neighboring method [2], based on the Gauss-Newton-Raphson algorithm. The software was implemented using the Open-Source EIDORS Library [23].

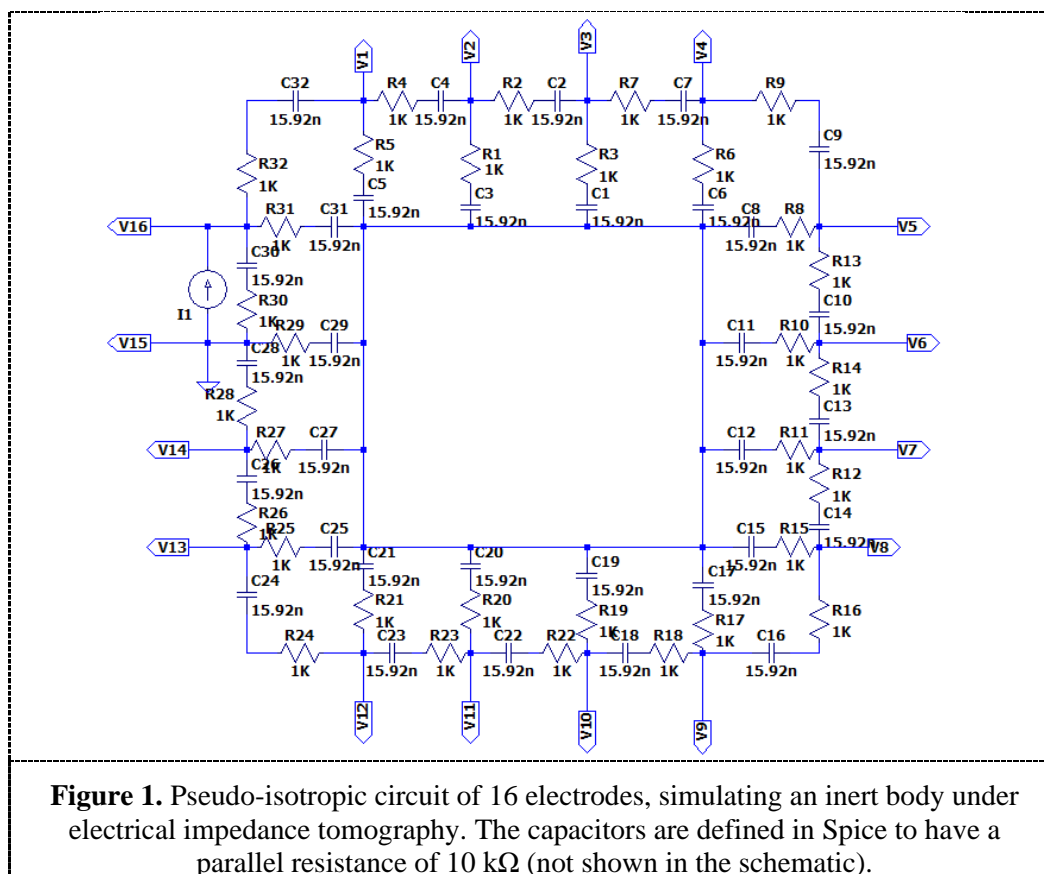
Traditionally, the tomographic image is reconstructed for a specific frequency [2–4]. Generally, the phasorial amplitude of the measured voltage is used to build the input matrix for the reconstruction algorithm. Our approach differs from the traditional use in filling the input matrix with the voltage values at different times (after proper fitting to the analytical expressions from section 2.3), rendering a time-domain set of images, which we called Step-response Electrical Impedance Tomography (srEIT).

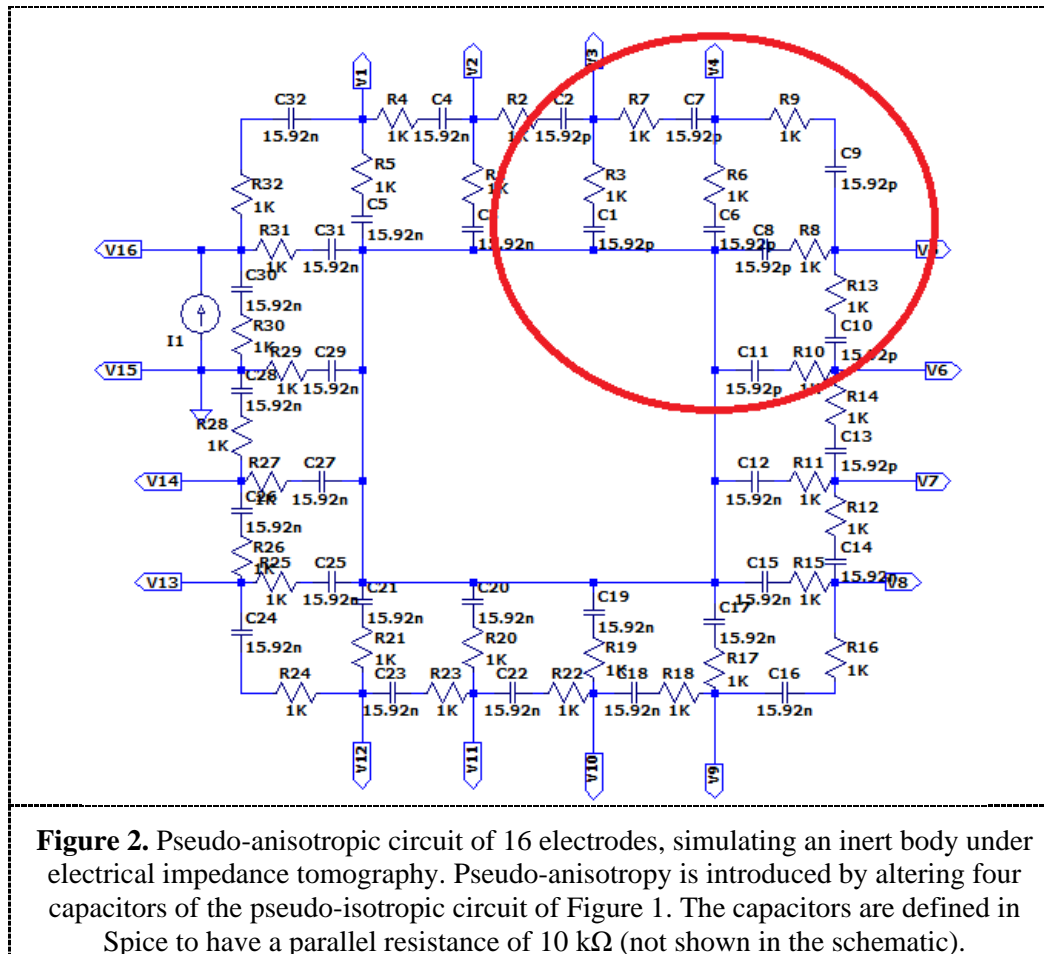
### 2.5. System simulations

For a proof-of-concept, we built two network-circuits from resistors and capacitors. The first circuit is an pseudo-isotropic circuit formed by Lopicque impedances of 1 k $\Omega$  resistors and 15.92 nF capacitors with a parallel resistance of 10 k $\Omega$  (see Figure 1). The characteristic time of each Lopicque impedance in this circuits is  $\tau \approx 14.47 \mu\text{s}$ .

The second circuit is pseudo-anisotropic. The eight capacitors in the right upper side have been changed from 15.92 nF to 15.92 pF, while preserving their parallel resistance (see Figure 2).

The measurement process consists of connecting a step current source between two points of the circuit while measuring the voltage in the other 14 points. This process is repeated 16 times by moving the connection points of the current source, according to the neighboring method. The voltages were extracted at four times:  $0.5\tau$ ,  $\tau$ ,  $2\tau$ , and  $100\tau$ . The simulations were performed using LTSpice XVII. The simulation circuits are available in the repository [24].





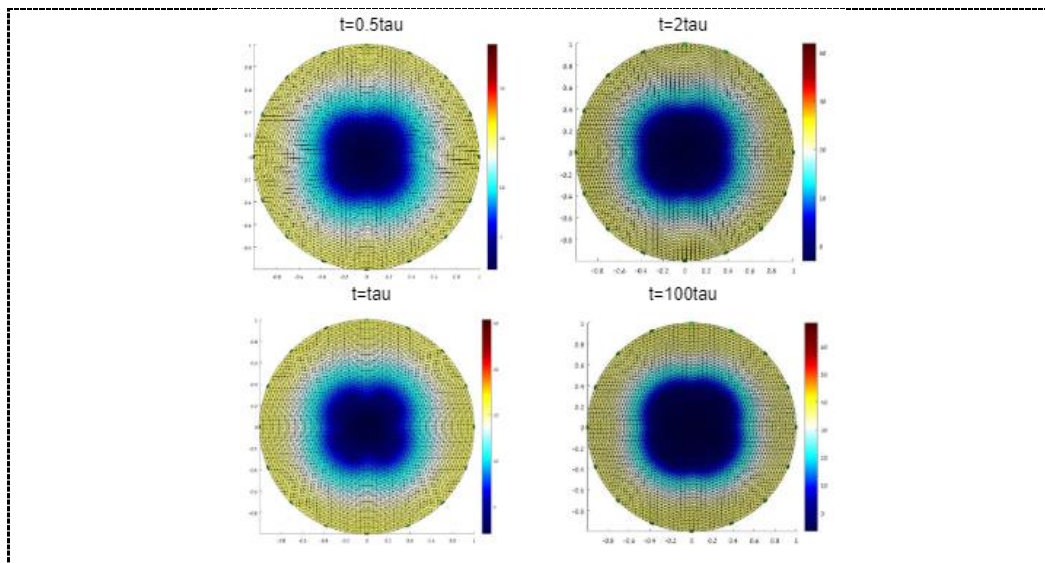
### 3. Results

For each circuit, the reconstruction process is done at four different times of the time constant:  $0.5\tau$ ,  $\tau$ ,  $2\tau$ , and  $100\tau$ , which is steady-state for the pseudo-isotropic circuit, where (see section 2.5)  $\tau \approx 14.47 \mu\text{s}$ .

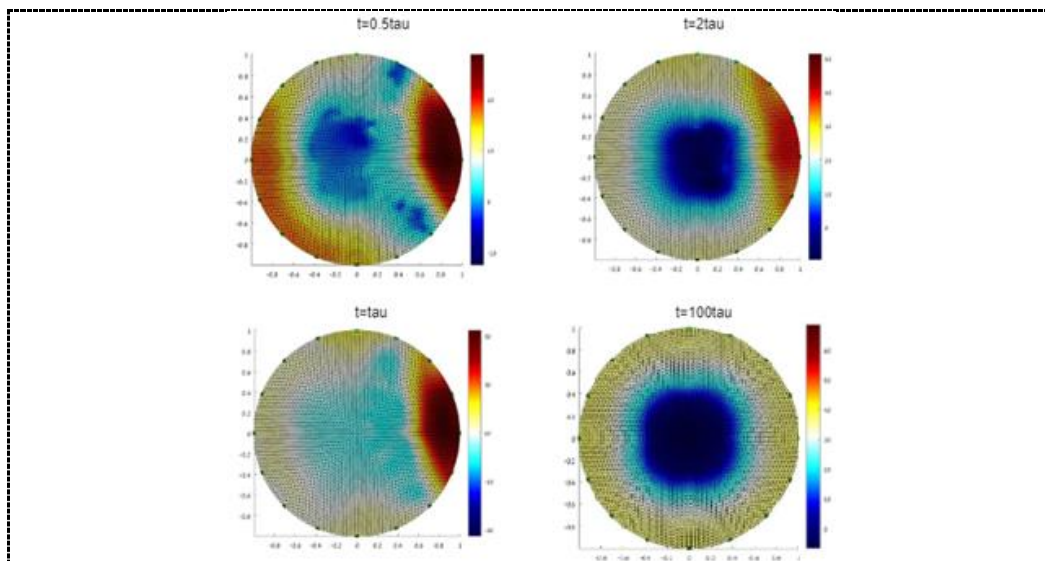
The four reconstructed images in Figure 3 show a uniform pattern of impedance distribution at all times, as expected from the pseudo-isotropic circuit. It is important to note that the color distribution is normalized per image.

Figure 4 shows the four reconstructed images of the pseudo-anisotropic network. Again, the color distribution is normalized per image. The anisotropic nature of the reconstructed images follows the pseudo-anisotropy of the circuit itself.

The difference between the circuits is in the capacitances' values; however, all resistances (in series and in parallel to each capacitor) are kept the same. Thus, in the steady state of the step response (DC) almost no difference in reconstructed images would be observed.



**Figure 3.** Images reconstructed from the pseudo-isotropic network at  $0.5\tau$ ,  $\tau$ ,  $2\tau$ , and  $100\tau$ , where  $\tau \approx 14.47 \mu\text{s}$ .



**Figure 4.** Images reconstructed from the pseudo-anisotropic network at  $0.5\tau$ ,  $\tau$ ,  $2\tau$ , and  $100\tau$ , where  $\tau \approx 14.47 \mu\text{s}$ .

#### 4. Discussion and Conclusions

Time-domain step-response analysis of a system (particularly a bioimpedance) presents the main advantage of being fast and non-time consuming when performing a broadband study. All the spectral information of interest is measured in a single sweep. This feature is particularly important when dealing with highly dynamic biological systems or for fast feedback in medical procedures, for which its high precision does not override the time-consumption drawback of traditional frequency-domain techniques.

However, a significant drawback should be noted: the method is more sensitive to the noise induced by jitter and high-speed ADC quantization [25]. The effects of the noise can be overcome by

performing several measurements and averaging (jeopardizing the advantage of non-time consuming) or by fitting to known analytical functions reviewed in this paper.

As Pliquett exposed in several works [26–28], sophisticated data processing and high-quality hardware design, together with experience, are fundamental to designing a time-domain system whose precision is as good as traditional frequency-domain systems with the clear advantage of the speed of measurement. Finally, the balance between the advantages and disadvantages of this technique is a delicate interplay between the system of interest and the application requirements.

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